EXERCISE – IV

ADVANCED SUBJECTIVE QUESTIONS

1. If , E = $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ and F = $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ calculate the matrix product EF & FE and show that $E^2F + FE^2 = E$.

2. Find the number of 2 × 2 matrix satisfying
(i) a_{ij} is 1 or –1

(ii) $a_{11}^2 + a_{12}^2 = a_{21}^2 + a_{22}^2 = 2$

(iii) $a_{11}\, a_{21} + a_{12}\, a_{22} = 0$

3. Find the value of x and y satisfy the equations

$$\begin{bmatrix} 3 & -2 \\ 3 & 0 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} y & y \\ x & x \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3y & 3y \\ 10 & 10 \end{bmatrix}$$

4. Prove that the product of two matrices, $\begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ & $\begin{bmatrix} \cos^2\phi & \sin\phi\cos\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ is a null matrix when θ & ϕ differ by an odd multiple of $\pi/2$.

5. Define A = $\begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$. find a vertical vector V such that $(A^8 + A^6 + A^4 + A^2 + I)V = \begin{bmatrix} 0 \\ 11 \end{bmatrix}$
(where I is the 2 × 2 identity matrix).

6. If, A = $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that the matrix A is a root of the polynomial $f(x) = x^3 - 6x^2 + 7x + 2$.

7. If the matrices A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (a, b, c, d not all simultaneously zero) commute, find the value of $\frac{d-b}{a+c-b}$. Also show that the matrix which commutes with A is of the form $\begin{bmatrix} \alpha-\beta & 2\beta/3 \\ \beta & \alpha \end{bmatrix}$.

8. If $\begin{bmatrix} a & b \\ c & 1-b \end{bmatrix}$ is an idempotent matrix. Find the value of f(a), where $f(x) = x - x^2$, when bc = 1/4. Hence otherwise evaluate a.

9. If the matrix A is involutary, show that $\frac{1}{2}(I + A)$ and $\frac{1}{2}(I - A)$ are idempotent and $\frac{1}{2}(I + A).\ \frac{1}{2}(I - A) = O$

10. If A = $\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is an orthogonal matrix, find the values of α, β, γ.

11. Given matrices A=$\begin{bmatrix} 1 & x & 1 \\ x & 2 & y \\ 1 & y & 3 \end{bmatrix}$; B=$\begin{bmatrix} 3 & -3 & z \\ -3 & 2 & -3 \\ z & -3 & 1 \end{bmatrix}$ Obtain x, y and z if the matrix AB is symmetric.

12. Let X be the solution set of the equation $A^x = I$, where A = $\begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ and I is the corresponding unit matrix and $x \subseteq N$ then find the minimum value of $\sum (\cos^x\theta + \sin^x\theta)$, $\theta \in R$.

13. Prove that $(AB)^T = B^T . A^T$, where A & B are conformable for the product AB. Also verify the result for the matrices, A = $\begin{bmatrix} 1 & 2 \\ 2 & -3 \\ -1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & -3 & 5 \\ 1 & 2 & 3 \end{bmatrix}$.

14. Express the matrix $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & -6 \\ -1 & 0 & 4 \end{bmatrix}$ as sum of a lower triangular matrix & an upper triangular matrix with zero in its leading diagonal. Also Express the matrix as a sum of a symmetric & a skew symmetric matrix.

15. A is a square matrix of order n.
ℓ = maximum number of distinct entries if A is a triangular matrix.
m = maximum number of distinct entries A is a diagonal matrix.
p = minimum number of zeros if A is a triangular matrix
If $\ell + 5 = p + 2m$, find the order of the matrix

16. Consider two matrices A and B where $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$;

$B = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$. If n (A) denotes the number of elements in A such that n (XY) = 0, when the two matrices X and Y are not conformable for multiplication.
If C = (AB)(B′A); D = (B′A) (AB) then, find the value of

$\left(\frac{n(C)(|D|^2 + n(D))}{n(A) - n(B)}\right)$.

17. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then prove that value of f and g satisfying the matrix equation $A^2 + fA + gI = O$ are equal to $-t_r(A)$ and determinant of A respectively. Given a, b, c, d are non zero reals and

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

18. $A_{3 \times 3}$ is a matrix such that | A | = a, B = (adj A) such that | B | = b. Find the value of $(ab^2 + a^2b + 1)$ S

where $\frac{1}{2}S = \frac{a}{b} + \frac{a^2}{b^3} + \frac{a^3}{b^5} + \ldots\ldots$ up to ∞, and a = 3.

19. For the matrix $A = \begin{bmatrix} 4 & -4 & 5 \\ -2 & 3 & -3 \\ 3 & -3 & 4 \end{bmatrix}$ find A^{-2}.

20. Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ find P such that

$BAP = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

21. Find the inverse of the matrix :

(i) $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w^2 & w \end{bmatrix}$ where w is the cube root of unity.

(iii) $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

22. Show that,

$$\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

23. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then show that

F(x). F(y) = F(x + y). Hence prove that $[F(x)]^{-1} = F(-x)$.

24. If A is a skew symmetric matrix and I + A is non singular, then prove that the matrix $B = (I - A)(I + A)^{-1}$ is an orthogonal matrix. Use this to find a matrix B given $A = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$.

25. Use matrix to solve the following system of equations.

(i)
$$\begin{aligned} x + y + z &= 3 \\ x + 2y + 3z &= 4 \\ x + 4y + 9z &= 6 \end{aligned}$$

(ii)
$$\begin{aligned} x + y + z &= 6 \\ x - y + z &= 2 \\ 2x + y - z &= 1 \end{aligned}$$

(iii)
$$\begin{aligned} x + y + z &= 3 \\ x + 2y + 3z &= 4 \\ 2x + 3y + 4z &= 7 \end{aligned}$$

(iv)
$$\begin{aligned} x + y + z &= 3 \\ x + 2y + 3z &= 4 \\ 2x + 3y + 4z &= 9 \end{aligned}$$

26. Given that $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 3 \\ 1 & -1 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 10 \\ 13 \\ 9 \end{bmatrix}$

and that Cb = D. Solve the matrix equation Ax = b.

27. Find the matrix A satisfying the matrix equation,

$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} . A . \begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix}$.

28. If $A = \begin{bmatrix} k & m \\ l & n \end{bmatrix}$ and $kn \neq lm$; then show that

$A^2 - (k + n) A + (kn - lm) I = O$. Hence find A^{-1}.

29. Given $A = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 9 & 3 \\ 3 & 1 \end{bmatrix}$. I is a unit matrix of order 2. Find all possible matrix X in the following cases.

(i) AX = A **(ii)** XA = I **(iii)** XB = O but BX ≠ O.

30. Find the product of two matrices A & B, where

$A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve

the following system of linear equations
$x + y + 2z = 1$; $3x + 2y + z = 7$; $2x + y + 3z = 2$.

31. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ then, find a non–zero square matrix X of order 2 such that AX = O. Is XA = O.

If $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, is it possible to find a square matrix X such that AX = O. Give reasons for it.

32. Determine the value of a and b for which the

system $\begin{bmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b \\ 3 \\ -1 \end{bmatrix}$

(i) has a unique solution; **(ii)** has no solution and **(iii)** has infinitely may solutions

33. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$; $B = \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix}$; $C = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ and $X = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$

then sole the following matrix equation.

(a) AX = B = I **(b)** (B – I) X = IC **(c)** CX = A

34. If A is an orthogonal matrix and B = AP where P is non singular matrix then show that the matrix PB^{-1} is also orthogonal.